Volatility in Istanbul Stock Exchange

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Abstract

Since economic agents make the decisions based on the perceived distribution of the random variables in the future, assessment and measurement of the variance has a significant impact on their course of action. Therefore, market participants’ ability to accurately measure and predict the stock market volatility has wide spread implications. This capability has a particular importance in an environment, where the perception of high levels of volatility has the potential to erode the investor confidence and divert the capital inflows from equity markets. This is a particular concern for the emerging equity markets that lack the advanced institutional and informational infrastructures and which are very vulnerable to domestic and foreign capital flows. The purpose of this study is to determine the time-varying characteristics of volatility in an emerging stock market by utilizing rich family of ARCH models. The primary focus of the study is to explore the nature of volatility in the ISE.
1-Introduction

Since economic agents make decisions based on the perceived distribution of the random variables in the future, assessment and measurement of the variance has a significant impact on their course of action. Therefore, market participants’ ability to accurately measure and predict the stock market volatility has wide spread implications. This capability has particular importance in an environment, where the perception of high levels of volatility has the potential to erode the investor confidence and divert the capital inflows from equity markets. This is a particular concern for the emerging equity markets that lack the advanced institutional and informational infrastructures and very vulnerable to domestic and foreign capital flows.

Early studies of the stock return behavior based on the constant variance assumption, traditionally has neglected the time varying nature of stock return variability. However, recent investigations of time-series properties of stock returns relaxed the implausible assumption of constant variance, and concentrated in models describing time-varying variance. These models, based on Autoregressive Conditional Heteroscedastic (ARCH) model of Engel (1982), have been popular in a wide range of financial applications and continuously improved with new generations of models. ARCH modeling can account for volatility clustering, that is the tendency of large stock price changes to be followed by large stock price changes. Empirical evidence suggests that variance does not only change over time, but it also evolves in a predictable pattern that is detectable in the immediate history of the process. Generalized ARCH models (GARCH) introduced by Bollersev (1986) attempts to formulate these patterns in stock return data. From a theoretical point of view these models present linearity which is a crucial property since they imply an ARMA equation for the squared innovation process, which allows for a complete study of the distributional properties of the innovation process. It also simplifies the statistical inference. In addition to an adequate model of dependence of volatility, GARCH models take into account fat-tailed distribution of the stock returns. On the other hand, GARCH models contain several limitations. GARCH models imply that past values of the innovation on the current volatility is only a function of their magnitude. However, it is argued that this feature is generally not true in the financial context (Nelson, 1990). Typically, volatility tends to be higher after a decrease than after an equal increase (Campbell and Hentsheel, 1990; Christie 1982; Nelson
1990b, 1991; Schwert 1989). Naturally the symmetric conditional variance can not capture such phenomena. A new generation of models EGRACH (Nelson 1991), TGARCH (Rabemananjara and Zakoian, 1993), C-ARCH, CA-ARCH, are used to formulate negative asymmetry in volatility.

The GARCH model proposed by Engle (1982) also contributed to the study of the relationship between market risk and expected returns. The GARCH-M model introduced by Engle, Lilien and Robins (1987) explicitly links the conditional variance to the conditional mean of returns. The conditional mean of returns are specified as a function of past returns and a function that links the conditional variance to the conditional mean. Typically functional forms of conditional variance include linear, square-root and logarithmic. In the context of this study, we tested the relationship between the conditional variance and the conditional mean by using alternative functional forms.

The purpose of this study is to determine the time-varying characteristics of volatility in an emerging stock market by utilizing rich family of ARCH models. The primary focus of the study is to explore the nature of volatility in ISE. More specifically the study is designed to answer following questions:

- Does volatility change in ISE over time? If it does change, is there a predictable pattern in this change (Is conditional heteroskedasticity significant or is volatility clustering present?)
- What are the relative impact of positive and negative shocks in the market on the volatility (Is there any evidence of negative asymmetry observed in developed stock markets?).
- What are the relative impact of transitory and permanent components on the volatility?
- Is there a positive risk-return trade-off in ISE returns? (Do estimated conditional variances contribute to the mean model estimation?)

The study will progress in the following format: Section two will introduce and review the basic features of the ARCH models used in this study. Section three will briefly discuss the data and volatility in Istanbul Stock Exchange. Section four will present the empirical results, and finally section five will present the concluding remarks.
2- Review of GARCH Models

Empirical studies in 1960 and 1970s identified a number of characteristics commonly observed in stock returns. Included in these are serial correlation in successive returns and squared returns, distinct periods of volatility and stability, negative asymmetry and clustered observations around the mean and the tails (leptokurtic). These peculiarities at the time were dealt with ARIMA modeling. The ARIMA models are based on the assumption that the disturbance terms have constant variance. However, time varying nature of the variance of the disturbance term had severe practical implications and undermined the value of forecasts generated by the ARIMA models.

2.1. ARCH Model

Since effective modeling of stock return demands accurate representation of the variance component, a new generation of models that account for empirical peculiarities of data were needed. ARCH models developed by Engels (1982) served this purpose. They allowed volatility clustering or distinct periods of high volatility and stability in successive periods.

ARCH models essentially consists of two linked equations: the mean equation and the variance equation: The mean equation can be a standard ARIMA \((m,d,n)\) process. In its simplest form ARCH(1) the mean and the variance equations can be expressed as

\[
Y_t = \phi Y_{t-1} + \epsilon_t
\]

\[
h_t = \omega + \alpha \epsilon_{t-1}^2
\]

The variance equation expresses how the variance changes over time and includes squared lagged disturbances. The generic ARCH\((q)\) process with \(q\) lags can be expressed as:

\[
h_t = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{i-1}^2
\]

2.2. GARCH\((p,q)\) Model

Some restrictive attributes of ARCH models such as imposing a fixed lag structure to avoid negative parameter estimates, led to search for a new and general class of processes.
Bollersev (1986) introduced the Generalized Autoregressive Heteroscedastic (GARCH) model. This model was more flexible in its lag structure and integrated past conditional variances as well as past squared disturbances of ARCH processes. The GARCH($p,q$) model with conditional variance function is commonly expressed as

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i}, \alpha_i \geq 0; \beta_i \geq 0$$

This extension of ARCH model is very much like the extension of AR models to ARMA models (Bollersev, 1986), and it is argued that it allows a more parsimonious description of time series. In the ARCH($q$) process, the conditional variance is specified as a linear function of only past sample variances, whereas the GARCH ($p,q$) process allows lagged conditional variances to enter the model. Bollersev (1986) argues that incorporation of past conditional variances correspond to some sort of an adaptive learning process.

Another important attribute of GARCH($p,q$) processes is their leptokurtic distribution, which concurs with the empirical characteristics of stock return data.

2.3. Exponential GARCH($p,q$) (EGARCH) Model

Although GARCH models satisfactorily accounted for most of the empirical features of the financial time series, a commonly observed characteristic, negative asymmetry was not successfully captured by GARCH models. The Exponential GARCH (EGARCH) model proposed by Pagan and Schwert (1990) and Nelson (1991) incorporated the observed asymmetry in stock return data. EGARCH($p,q$) process specified as

$$f(z_t) = \phi z_t + \gamma \left[ |z_t| - E(|z_t|) \right], \text{ where } z_t = \frac{\varepsilon_t}{h_t^{1/2}}$$

The advantage of using an exponential form for the conditional variance function $h_t$ is that the variance is positive for all choices of the parameters of the EGARCH process. The EGARCH model replaces lagged squared residuals in the GARCH model with a function which allows the model to account for asymmetry.

$$f(z_t) = \phi z_t + \gamma \left[ |z_t| - E(|z_t|) \right]$$
For $\phi < 0$, the term $\phi z_t$ induces asymmetry in the model. The second term in the function $\gamma[|z_t| - E(|z_t|)]$ accounts for the magnitude. Residuals that are greater in magnitude than expected have a positive effect on the conditional variance. The residuals enter the model as standardized residuals with respect to current volatility. This allows the model to take into account of extreme residuals rather than the relatively moderate residuals that occur during a period of high volatility.

2.4. Threshold GARCH (TGARCH) Model

TGARCH Model is the product of another attempt to account for asymmetry in volatility (Engels and Bollersev, 1986; Zakoian, 1990). In this model, conditional variance is a piecewise function, thereby allowing different reactions of volatility to different signs and magnitudes of shock. The model is built by including a new term as a dummy variable which takes the value 1, when the news is bad., i.e. when $\epsilon_t < 0$, and zero otherwise. If the coefficient of the new term is significant, the ARCH effect on the conditional variance is augmented. This is consistent with the higher volatility associated with a bad news. The model is specified as follows:

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i e_{t-i}^2 + \sum_{i=1}^{p} \gamma_i e_{t-i}^2 d_{t-1} + \sum_{i=1}^{p} \beta_i h_{t-i}$$

$$d_{t-1} = 1 \text{ if } \epsilon_t < 0 \text{ , and } d_{t-1} = 0 \text{ otherwise}$$

2.5. Component ARCH (C-ARCH) Model

The component ARCH model modifies the constant (long term) component of the conditional variance into a time-varying component. This modification allows us to measure the rate at which short term variance converges to long term variance as well as the significance of transitory and permanent ARCH and GARCH effects in the conditional variance. The C-ARCH(1,1) model can be derived by substituting $\omega$ with $q_t$ in the original GARCH(1,1) model, where $q_t$ is time-varying permanent component of conditional variance.
\[ h_t = q_t + \alpha (\varepsilon^2_{t-1} - q_{t-1}) + \beta (h_{t-1} - q_{t-1}) \]
\[ q_t = w + \rho (q_{t-1} - w) + \phi (\varepsilon^2_{t-1} - h_{t-1}) \]

This substitution yields the following model:

\[ h_t = w + \rho (q_{t-1} - w) + \phi (\varepsilon^2_{t-1} - \sigma^2_{t-1}) + \alpha (\varepsilon^2_{t-1} - q_{t-1}) + \beta (\sigma^2_{t-1} - q_{t-1}) \]

The coefficient \( \rho \) measures the rate at which \( q_t \) converges to \( w \). The coefficient \( \phi \) measures the permanent combined ARCH-GARCH effect. While \( \beta \) measures the transitory GARCH effect, \( \alpha \) measures the transitory ARCH effect.

### 2.6. Asymmetric Component ARCH Model (AC-ARCH)

A simple extension of C-ARCH model generated the AC-ARCH which interjects a dummy variable to account for asymmetry in the C-ARCH model. This simple modification decomposes the transitory ARCH effect, and allows us to detect leverage effect in the transitory component of conditional variance. The model can be specified as follows:

\[ h_t = w + \rho (q_{t-1} - w) + \phi (\varepsilon^2_{t-1} - \sigma^2_{t-1}) + \alpha (\varepsilon^2_{t-1} - q_{t-1}) + \gamma (\varepsilon^2_{t-1} - q_{t-1})d_{t-1} + \beta (\sigma^2_{t-1} - q_{t-1}) \]

\( d_{t-1} = 1 \) if \( \varepsilon_t < 0; \) 0 otherwise

### 3. Data and Study

The objective of this study was set to explore the nature of volatility in ISE returns. Data used in this study consists of the daily value of ISE Index from January 1986 to December 1996. The index is a weighted average based on individual stock closing prices for a select group of stocks quoted at the Istanbul Stock Exchange. The analysis here focused on the return series defined as the first difference of natural logarithm of the price. Due to
unavailability of the information for the entire period that the study covers, the returns used in this study do not include dividends and returns refer only capital gains.

The first step of the study was to estimate a mean model for ISE returns. The error terms of the mean model was used to analyze the ARCH and GARCH effects. The relationship between conditional variance and the conditional mean (the contribution of the conditional variance to the mean model) was tested for GARCH, TGARCH, EGARCH, C-ARCH and CA-ARCH models. was tested for each conditional volatility model. The mean models are referred as GARCH-M, TGARCH-M, EGARCH-M etc. The coefficients of the conditional variance functions (linear and squared functions only since coefficients of the logarithmic functions proved to be insignificant in all variance models) were reported in respective tables.

4. Empirical Results

The level ISE returns was modeled as ARIMA(0,1,1) (R) (or ARMA(0,1)D(R)). In other words the difference of the return series $R_t$ is modeled as a drift coupled with the first moving average term. The residuals of the mean model

$$R_t = \alpha + \phi R_{t-1} + \epsilon_t$$

was used to model conditional variance of ISE returns.

The first model is GARCH(1,1). The following model was estimated and the contribution of estimated conditional variance and standard deviation to the mean model was tested.

$$h_t = w + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$
Table-1: GARCH Model

<table>
<thead>
<tr>
<th>Variance Model</th>
<th>Mean Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>β</td>
</tr>
<tr>
<td>GARCH(1,1)</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>(8.34)</td>
</tr>
<tr>
<td>GARCH(1,1)-M-S</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>(8.34)</td>
</tr>
<tr>
<td>GARCH(1,1)-M-V</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>(8.34)</td>
</tr>
</tbody>
</table>

The analysis of GARCH(1,1) model indicates that lagged conditional variance and lagged squared disturbance has an impact on the conditional variance. Including distant lags did not improve the model. The coefficients of the conditional variance or standard deviation in the mean model proved to be significant. In other words, incorporating volatility estimates into the mean model, improves the estimation of the mean returns.

In order to test for the asymmetry in the volatility of ISE returns, two alternative models were used: TGARCH and EGARCH. In the TGARCH model, the impact of the bad news was designed to be quadratic. The model was specified as:

\[ h_t = w + \alpha \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1}^2 + \beta h_{t-1}, \quad d_{t} \begin{cases} 1 & \text{if } \varepsilon_{t} < 0; \\ 0 & \text{otherwise} \end{cases} \]

The significance of \( \gamma \) indicates that the effect of bad news is larger on the volatility than the good news. In other words, the significance of the dummy coefficient implies the leverage effect in ISE returns.
Table-2: TARCH Model

<table>
<thead>
<tr>
<th>Variance Model</th>
<th>Mean Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \alpha )</td>
</tr>
<tr>
<td>\text{TARCH}(1,1)</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>(6.49)</td>
</tr>
<tr>
<td>\text{TARCH}(1,1)-M-S</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>(6.55)</td>
</tr>
<tr>
<td>\text{TARCH}(1,1)-M-V</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>(6.57)</td>
</tr>
</tbody>
</table>

The model verifies the significance of ARCH and GARCH effects in the conditional variance of ISE returns. However, the asymmetry is not verified in ISE returns. Namely, the model does not provide any support the proposition that the bad news increases the volatility more than the good news. Incorporation of the TARCH estimation of conditional variance and standard deviation improved the mean model as in the GARCH model.

An alternative model that accounts for asymmetry is the EGARCH model of Bollerslev. In this model the impact of the variance is exponential and residuals are standardized with respect to current volatility to account for relatively large unexpected changes in returns over a volatility cycle. The EGRACH(1,1) model used in this study is specified as follows:

\[
\log(h_t) = w + \alpha \frac{\varepsilon_t - 1}{\sigma_t - 1} + \gamma \frac{\varepsilon_t - 1}{\sigma_t - 1} + \beta \log(h_t - 1)
\]

Table-3: EGARCH Model

<table>
<thead>
<tr>
<th>Variance Model</th>
<th>Mean Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \alpha )</td>
</tr>
<tr>
<td>\text{EGARCH}(1,1)</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>(9.50)</td>
</tr>
<tr>
<td>\text{EGARCH}(1,1)-M-S</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>(9.61)</td>
</tr>
<tr>
<td>\text{EGARCH}(1,1)-M-V</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>(9.72)</td>
</tr>
</tbody>
</table>
The results of the EGARCH analysis concur with the TARCH results. While the ARCH and GARCH effects are significant, asymmetry can not be confirmed. This result implies that bad news have no larger impact on the volatility than the good news. Incorporation of the conditional variance and the deviation improved the mean model performance in this particular case as well.

Although the results of TARCH and EGARCH analysis provide no evidence of leverage effect in ISE returns, a decomposition of permanent and transitory components of volatility may provide further insight on the nature of volatility in ISE returns. We used Component ARCH (C-ARCH) model to decompose permanent and transitory volatility. The model used in this analysis was specified as

\[
h_t = w + \rho(q_{t-1} - w) + \phi(\varepsilon_t^2 - 1 - \sigma_t^2 - 1) + \alpha(\varepsilon_t^2 - q_{t-1}) + \beta(\sigma_t^2 - q_{t-1})
\]

Table 4: C-ARCH Model

<table>
<thead>
<tr>
<th>Model</th>
<th>Variance Model</th>
<th>Mean Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( w )</td>
<td>( \rho )</td>
</tr>
<tr>
<td>C-ARCH(1,1)</td>
<td>0.00</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>(2.06)</td>
<td>(9.50)</td>
</tr>
<tr>
<td>C-ARCH(1,1)-M-S</td>
<td>0.00</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>(4.99)</td>
<td>(9.61)</td>
</tr>
<tr>
<td>C-ARCH(1,1)-M-V</td>
<td>0.00</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>(2.25)</td>
<td>(9.72)</td>
</tr>
</tbody>
</table>

The results of the analysis indicate that transitory ARCH-GARCH effect is significantly larger than permanent ARCH-GARCH effect. While the coefficients of the transitory components sum up to 0.83, the coefficient of the permanent component is 0.07. In other words, short term dynamics dominate the conditional variance. The coefficient \( \rho \) measures the rate at which \( q_t \) (time varying component) reverts to \( w \) (permanent component). The estimated value of \( \rho \) is 0.99 which implies that the reversion is very slow in ISE case.
Finally, we searched for a possible asymmetric effect on the conditional variance by decomposing the transitory component into two parts as in the TARCH model. A dummy variable was incorporated into the C-ARCH model. The dummy variable $d_{t-1}$ is 1 if $\varepsilon < 0$ and, 0 otherwise. If a decline in the index tend to increase the volatility, the coefficient of the transitory component with a dummy variable is expected to be significantly different than 0, if not it is expected to be insignificant.

$$\sigma_t^2 = w + \rho(q_{t-1} - w) + \phi(\varepsilon_{t-1}^2 - \sigma_{t-1}^2) + \alpha(\varepsilon_{t-1}^2 - q_{t-1}) + \gamma(\varepsilon_{t-1}^2 - q_{t-1})d_{t-1} + \beta(\sigma_{t-1}^2 - q_{t-1})$$

Table 4: AC-ARCH Model

<table>
<thead>
<tr>
<th>Variance Model</th>
<th>Mean Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$w$</td>
</tr>
<tr>
<td>$AC$-$ARCH(1,1)$</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(1.86)</td>
</tr>
<tr>
<td>$AC$-$ARCH(1,1)$-$M$-$S$</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(1.82)</td>
</tr>
<tr>
<td>$AC$-$ARCH(1,1)$-$M$-$V$</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(2.00)</td>
</tr>
</tbody>
</table>

Our findings indicate that transitory component of the conditional variance does not submit any evidence of an asymmetric response to bad news relative to good news. Although this result does not collaborate with the empirical finding reported in the literature, it is consistent in the context of this study, where we could not find any evidence of asymmetric effect in volatility.

5- Concluding Remarks

The analysis of Istanbul Stock Exchange Index returns confirms time variation in stock market volatility. Our findings indicate that conditional variance of ISE returns are significantly affected by lagged shocks and the lagged variance contains information about the current volatility. In other words our findings confirm the well documented volatility clustering in stock returns. Another widely discussed empirical characteristic of the stock returns is the negative asymmetry in volatility. There is ample empirical evidence that a
shock associated with a bad news tend to trigger a higher increase in volatility than the good news. However, our findings did not confirm this widely observed empirical characteristic for ISE returns. News impact curve for ISE remains to be symmetric, and GARCH(1,1) adequately models the volatility in ISE. This result concurs the findings of Koutmos (1992) which reported lack of asymmetry in Canadian, French, Japanese and Dutch stock returns, and positive asymmetry for Australian stock returns. Our findings support the argument that the negative asymmetry is not a universal phenomena. The non-homogenous response to volatility shocks can be interpreted as a factor contributing to the stability of the international financial markets. Since our data set includes only one emerging stock market, it is impossible to argue that symmetric news impact is an emerging market peculiarity. However, this finding motivates us to extend the context of the study to other emerging markets to explore the extent that this is an emerging market peculiarity. A confirmation of our result for other emerging markets has the potential to further justify the diversification benefits of investing in emerging markets.

Our analysis of short term and long term components of the ISE volatility indicates that the short term component in ISE volatility is significantly larger than the permanent component (long term) and the reversion to permanent volatility is very slow. The implication of this finding is that volatility in ISE is governed by short term volatility shocks, and the convergence of current volatility level to average volatility takes long time. This finding implies that the pricing inefficiencies are rather large to the extent that market participants use constant measures of volatility to price Turkish equities. This result has a particular importance in the context of this study, since our mean model estimates confirmed the positive relationship between conditional variance and the conditional mean. Incorporation of the linear (square root of conditional variance) and squared functions yields positive and significant coefficients, which implies positive risk premiums on the conditional volatility.
References