

Pattern Demansfrating Recursive Formula

$n=1$ (only 0 allowed)

1 $\{0, 0, 0, 0\}$

$n=2$ (1 and 0 allowed)

2 $\{1, 1, 1, 1\}, \{1, 1, 1, 0\}$

1 $\{1, 1, 0, 0\},$

1 $\{1, 0, 0, 0\}$

Repeated $\{0, 0, 0, 0\}$

$n=3$ (2, 1, and 0 allowed)

3 $\{2, 2, 2, 2\}, \{2, 2, 2, 1\}, \{2, 2, 2, 0\},$

2 $\{2, 2, 1, 1\}, \{2, 2, 1, 0\},$

1 $\{2, 2, 0, 0\}$

2 $\{2, 1, 1, 1\}, \{2, 1, 1, 0\}$

1 $\{2, 1, 0, 0\}$

1 $\{2, 0, 0, 0\}$

$\{1, 1, 1, 1\}$, etc: Repeating

$$h = 3$$

$$4 \quad \{3, 3, 3, 3\}, \{3, 3, 3, 2\}, \{3, 3, 3, 1\}, \{3, 3, 3, 0\},$$

$$3 \quad \{3, 3, 2, 2\}, \{3, 3, 2, 1\}, \{3, 3, 2, 0\},$$

$$2 \quad \{3, 3, 1, 1\}, \{3, 3, 1, 0\},$$

$$1 \quad \{3, 3, 0, 0\}$$

$$3 \quad \{3, 2, 2, 2\}, \{3, 2, 2, 1\}, \{3, 2, 2, 0\}$$

$$2 \quad \{3, 2, 1, 1\}, \{3, 2, 1, 0\},$$

$$1 \quad \{3, 2, 0, 0\}$$

$$2 \quad \{3, 1, 1, 1\}, \{3, 1, 1, 0\}$$

$$1 \quad \{3, 1, 0, 0\},$$

$$1 \quad \{3, 0, 0, 0\}$$

~~$\{2, 2, 2, 2\}$~~ Repeating

For just possibilities including 3:

$$(4+3+2+1) + (3+2+1) + (2+1) + (1)$$

$$= 4(1) + 3(2) + 2(3) + 1(4)$$

$$= \sum_{i=0}^n (n+i+1) \binom{n}{i} = \sum_{i=0}^n (ni - i^2 + i + n - i + 1)$$

$$= \sum_{i=0}^n -i^2 + ni + n + 1$$